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Abstract. The present study consists in studying the mean orbital motion of the CHAMP satellite, through a single long arc on a period of time of 200 days in 2001. We actually investigate the sensibility of its mean motion to its accelerometric data, as measures of the surface forces, over that period.

In order to accurately determine the mean motion of CHAMP, we use "observed" mean orbital elements computed, by filtering, from 1-day GPS orbits. On the other hand, we use a semi-analytical model to compute the arc. It consists in numerically integrating the effects of the mean potentials (due to the Earth and the Moon and Sun), and the effects of mean surfaces forces acting on the satellite. These later are, in case of CHAMP, provided by an averaging of the Gauss system of equations.

Results of the fit of the long arc give a relative sensibility of about 10^{-3}, although our gravitational mean model is not well suited to describe very low altitude orbits. This technique, which is purely dynamical, enables us to control the decreasing of the trajectory altitude, as a possibility to validate accelerometric data on a long term basis.

Key words. Mean orbital motion, accelerometric data

1 Introduction

Why studying the solely long period effects on orbits, and in particular for the CHAMP satellite? Among the advantages, we can quote the continuous description of the evolution of orbital parameters: over a given period of time, the computed orbital elements are adjusted on observations only at the initial epoch of the arc. That long period approach has been developed by the GRGS, "Groupe de Recherche en Géodésie Spatiale", for about more than 10 years, in the CODIOR software (Exertier et al., 1994).

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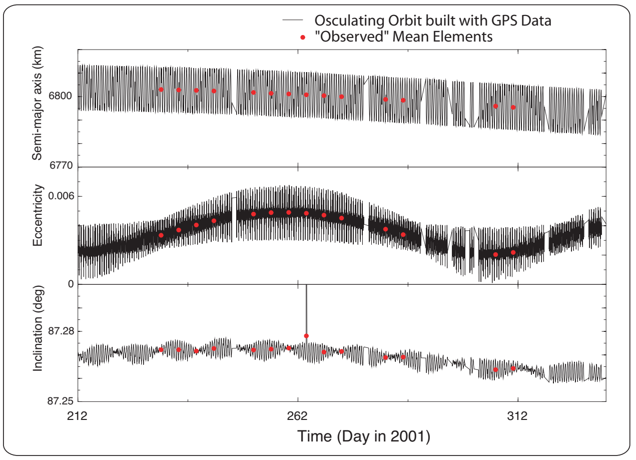

Fig. 1. The Observed Mean Elements (circles) deduced from the osculating orbits

We call "mean motion" the motion described by 6 orbital parameters for which all short periodic variations, linked to the period of revolution of the satellite, have been removed: the osculating differential equations of dynamics are transformed in an analytical way to get the averaged equations of dynamics (see Métris and Exertier, 1995). This transformation has been expressed by Métris (1991), for all the perturbations acting on the satellites, with respect to classical keplerian elements: the semi-major axis a, the eccentricity e, the inclination i, the ascending node Ω, the argument of perigee ω, and the mean anomaly M. To take into account the coupling effects between geodynamical parameters, an algorithm deduced from (Deprit, 1969) has been used, and computed with the algebric manipulator MS (Claes et al., 1988). Then the obtained averaged differential system verified by $\frac{da'}{dt}$, $\frac{de'}{dt}$, $\frac{di'}{dt}$, $\frac{d\Omega'}{dt}$, $\frac{d\omega'}{dt}$, and $\frac{dM'}{dt}$, is then integrated numerically. The integration step size varies from 12 hours for satellite with a high altitude such as LAGEOS, to 01:30 for CHAMP.

Table 1. "Observed" Mean Elements of CHAMP. (at $0^h\ 0^{min}\ 19^s$ for each date)

Day in 2001 (DOY)	$a\ (km)$	e	$i\ (rad)$	$\Omega\ (rad)$	$\omega\ (rad)$	$M\ (rad)$
25/05 (145)	6806.28221	$3.888580e-03$	$1.523132e+00$	$5.716833e-01$	$2.814128e+00$	$5.3630e+00$
30/05 (150)	6806.07340	$4.308192e-03$	$1.523125e+00$	$5.387671e-01$	$2.526129e+00$	$6.1088e-01$
09/06 (160)	6805.68345	$4.893797e-03$	$1.523094e+00$	$4.729402e-01$	$2.001149e+00$	$3.6857e+00$
14/06 (165)	6805.45877	$5.027619e-03$	$1.523100e+00$	$4.399983e-01$	$1.751527e+00$	$5.2434e+00$
19/06 (170)	6805.20278	$5.042196e-03$	$1.523090e+00$	$4.070647e-01$	$1.504430e+00$	$5.4073e-01$
14/07 (195)	6804.21183	$3.556922e-03$	$1.523080e+00$	$2.421985e-01$	$1.557047e-01$	$2.6393e+00$
19/07 (200)	6804.04726	$3.082365e-03$	$1.523063e+00$	$2.092147e-01$	$6.093751e+00$	$4.4463e+00$
24/07 (205)	6803.90098	$2.646128e-03$	$1.523088e+00$	$1.762229e-01$	$5.687903e+00$	$4.7583e-02$
29/07 (210)	6803.76524	$2.330767e-03$	$1.523087e+00$	$1.432553e-01$	$5.205031e+00$	$2.0243e+00$
03/08 (215)	6803.63319	$2.228741e-03$	$1.523090e+00$	$1.102704e-01$	$4.662077e+00$	$4.0749e+00$
08/08 (220)	6803.45822	$2.375377e-03$	$1.523118e+00$	$7.730452e-02$	$4.127783e+00$	$6.1328e+00$
13/08 (225)	6803.25792	$2.714502e-03$	$1.523113e+00$	$4.434866e-02$	$3.661787e+00$	$1.8597e+00$
18/08 (230)	6803.01890	$3.151972e-03$	$1.523125e+00$	$1.137314e-02$	$3.270249e+00$	$3.8187e+00$
23/08 (235)	6802.78114	$3.611237e-03$	$1.523143e+00$	$6.261611e+00$	$2.934493e+00$	$5.7478e+00$
28/08 (240)	6802.51174	$4.037581e-03$	$1.523132e+00$	$6.228651e+00$	$2.635556e+00$	$1.3831e+00$
12/09 (255)	6801.48869	$4.836343e-03$	$1.523138e+00$	$6.129779e+00$	$1.846483e+00$	$9.4686e-01$
17/09 (260)	6801.06083	$4.890453e-03$	$1.523156e+00$	$6.096809e+00$	$1.598467e+00$	$2.9620e+00$
27/09 (270)	6800.16159	$4.657645e-03$	$1.523126e+00$	$6.030825e+00$	$1.100856e+00$	$8.4366e-01$
02/10 (275)	6799.56850	$4.375503e-03$	$1.523120e+00$	$5.997823e+00$	$8.435786e-01$	$3.0177e+00$
07/10 (280)	6799.05970	$4.003218e-03$	$1.523086e+00$	$5.964788e+00$	$5.739729e-01$	$5.2659e+00$
12/10 (285)	6798.51899	$3.560818e-03$	$1.523088e+00$	$5.931724e+00$	$2.827565e-01$	$1.3078e+00$
17/10 (290)	6798.04108	$3.080134e-03$	$1.523061e+00$	$5.898661e+00$	$6.240625e+00$	$3.7206e+00$
01/11 (305)	6796.17552	$2.008292e-03$	$1.523006e+00$	$5.799295e+00$	$4.835287e+00$	$5.4768e+00$
06/11 (310)	6795.60040	$2.071759e-03$	$1.522998e+00$	$5.766118e+00$	$4.270222e+00$	$2.1025e+00$
01/12 (335)	6792.97150	$4.142496e-03$	$1.522963e+00$	$5.600058e+00$	$2.408330e+00$	$4.0049e+00$
06/12 (340)	6792.41668	$4.459608e-03$	$1.522976e+00$	$5.566818e+00$	$2.147855e+00$	$6.6662e-01$
11/12 (345)	6791.83220	$4.668439e-03$	$1.522969e+00$	$5.533574e+00$	$1.904352e+00$	$3.6575e+00$
16/12 (350)	6791.24413	$4.767613e-03$	$1.522987e+00$	$5.500310e+00$	$1.663690e+00$	$4.2466e-01$
21/12 (355)	6790.65526	$4.742050e-03$	$1.522999e+00$	$5.467064e+00$	$1.425274e+00$	$3.5360e+00$
26/12 (360)	6789.97174	$4.598800e-03$	$1.522988e+00$	$5.433801e+00$	$1.185720e+00$	$4.3238e-01$

Due to divisions by the eccentricity in $\frac{de'}{dt}$, $\frac{d\omega'}{dt}$, and $\frac{dM'}{dt}$, (Deleflie, 2002) has formulated the averaged differential system with respect to the non-singular elements for eccentricity $C = e\cos\omega$, $S = e\sin\omega$, and $\lambda = \omega + M$. For satellites with a small eccentricity such as CHAMP, this is in fact the equations verified by $\frac{dC'}{dt}, \frac{dS'}{dt}, \frac{d\lambda'}{dt}$ which are integrated in a numerical way, for gravitational effects as well as for non-gravitational effects.

We propose here to apply to the CHAMP orbit the method we have developed for geodetic satellites such as STARLETTE (Exertier et al., 1999) or AJISAI (Deleflie, 2002).

We transform in Sect. 2 the osculating orbits computed from May to December 2001 with GPS data to get the "observed" mean elements. These quantities are comparable to those computed with the CODIOR software.

Let note that a specific approach has been developed for CHAMP to build the second member of the averaged equations of dynamics verified by non-gravitational forces with the data obtained from the STAR accelerometer (Perret et al., 2001). This specific approach makes it possible to give mean characteristics of the STAR accelerometer, this is the subject of Sect. 3.

The long arc of CHAMP is shown in Sect. 4. Such a long period approach for CHAMP permits to give the characteristics of the orbital mean motion on the basis of observations obtained over a long period of time. And for example the initial conditions of the motion. These conditions are all the more determined with a great precision as there are few parameters which are used to compute the mean orbit.

The conclusion (Sect. 5) sums up the reached accuracy on the long arc of CHAMP, and presents what can now be developed to improve it.

2 The removal of short period effects from the observations

To adjust a mean orbit on measurements, it is necessary to get quantities comparable to those computed in the CODIOR software. We call these quantities *"observed" mean elements*, even if they are not the quantities effectively observed (this explaining the inverted comma).

There are two steps to build these "observed mean elements".

Table 2. Mean characteristics of the STAR accelerometer derived from CODIOR

	Limits of areas (DOY in 2001)	C_T	C_N	B_T $(m.s^{-2})$	B_N $(m.s^{-2})$
Aera 1	from 143 to 173	0, 42	0, 47	$-1,64\ 10^{-6}$	$-0,47\ 10^{-6}$
Area 2	from 173 to 226	0, 71	0, 25	$-2,6\ 10^{-6}$	$0,33\ 10^{-6}$
Area 3	from 226 to 281	0, 77	0, 25	$-2,8\ 10^{-6}$	$0,8\ 10^{-7}$
Area 4	from 281 to 293	0, 85	0, 25	$-3,2\ 10^{-6}$	$0,8\ 10^{-7}$
Area 5	from 293 to 341	0, 62	0, 25	$-2,3\ 10^{-6}$	$0,19\ 10^{-6}$

C_T: Tangential scale factor. C_N: Normal scale factor. B_T: Tangential bias. B_N: Normal bias.

First, short arcs are computed with a classical numerical integration: we use the GINS software to compute 1-day arcs, from the end of May 2001 to the end of December 2001, derived from on board GPS tracking data with the short perturbations being present. These orbits are formulated with orbital elements and not with positions and velocities, in order to isolate the short from the long period perturbations. The metric elements of all these orbits are shown in Fig. 1.

Secondly, all the short period terms are removed. This filtering approach, applied in the CANEL software, is described in (Exertier, 1990), and made up of two steps: first an analytical step based on an explicit formulation of the short period terms, expressed in a set of non singular elements for eccentricity. This formulation is deduced from an analytical solution of the Lagrange planetary equations (Deleflie et al., 2003). The osculating elements E_{osc} contain long as well as short period terms: $E_{osc} = E_{LP} + E_{SP}$. This solution formulates the short period terms E_{SP} on the basis of the osculating elements, terms which are removed from E_{osc}. This step enables to remove terms whose amplitude range from

decimeters to 10 km. Secondly, there is a numerical step based on a convolution product (Goad, 1977), to filter short terms not yet removed, in particular the resonances with a characteristic period of about a few days: 2,3 days in the case of CHAMP. That's why we have chosen a window width equal to 5 days. This width is not far from the Nyquist period, but allows to get enough "observed mean elements", about one every 5 days (see Table 1).

As a result, we give the filtered orbital parameters E_{LP} in Fig. 1.

3 Mean characteristics of the STAR accelerometer

In the CODIOR software, the equations of dynamics are formulated in a set of non singular elements for eccentricity: $a,\ C = e\cos\omega,\ i,\ \Omega,\ S = e\sin\omega,\ \lambda = \omega + M$. We use therefore the averaged Gauss equations to take into account the influence of a perturbative force $\overrightarrow{F}$, with respect to the corresponding mean variables (with a prime). This force is decomposed in the classical frame linked to the satellite, and oriented by the radial position: $\overrightarrow{F}\left|\begin{array}{l}\mathbf{R}\\ \mathbf{T}\\ \mathbf{N}\end{array}\right.$

Defining $\eta = \sqrt{1 - C'^2 - S'^2}$, and $\bar{n}$ the mean motion of the satellite (linked to the third law of Kepler $\bar{n}^2 a^3 = \mu$ where μ is the gravitational constant of the Earth), we can write (Deleflie, 2002):

$$\left|\begin{aligned}
\frac{da'}{dt} &= \frac{2\eta}{\bar{n}}\mathbf{T}\\
\frac{dC'}{dt} &= -\frac{S'\eta}{\bar{n}a'}\mathbf{R} + \frac{1}{\bar{n}a'}\left(-C'\eta - \frac{C'}{2} + \frac{C'(C'^2+S'^2)}{2(1+\eta)}\right)\mathbf{T} - \frac{3}{2}S'^2\frac{\cos i'}{\bar{n}a'\eta\sin i'}\mathbf{N}\\
\frac{di'}{dt} &= -\frac{3}{2}\frac{C'}{\bar{n}a'\eta}\mathbf{N}\\
\frac{d\Omega'}{dt} &= -\frac{3}{2}\frac{S'}{\bar{n}a\eta\sin i'}\mathbf{N}\\
\frac{dS'}{dt} &= \frac{C'\eta}{\bar{n}}\mathbf{R} + \frac{1}{\bar{n}a'}\left(-S'\eta - \frac{S'}{2} + \frac{S'(C'^2+S'^2)}{2(1+\eta)}\right)\mathbf{T} + \frac{3}{2}C'S'\frac{\cos i'}{\bar{n}a'\eta\sin i'}\mathbf{N}\\
\frac{d\lambda'}{dt} - \bar{n} &= \frac{1}{\bar{n}a'}(-3+\eta)\mathbf{R} + \frac{3}{2}S'\frac{\cos i'}{\bar{n}a'\eta\sin i'}\mathbf{T}
\end{aligned}\right. \tag{1}$$

The STAR accelerometer on board CHAMP measures, every second, the resultant acceleration due to non gravitational forces (air drag, Earth albedo, solar radiation, manoeuvers,...) in these three axes (radial, along-track and across-track), that is to say the quantities $\boldsymbol{R}_{\mathrm{STAR}}, \boldsymbol{T}_{\mathrm{STAR}}, \boldsymbol{N}_{\mathrm{STAR}}$. To take into account the instrumental characteristics, these quantities are corrected, in each axe, with a scale factor and

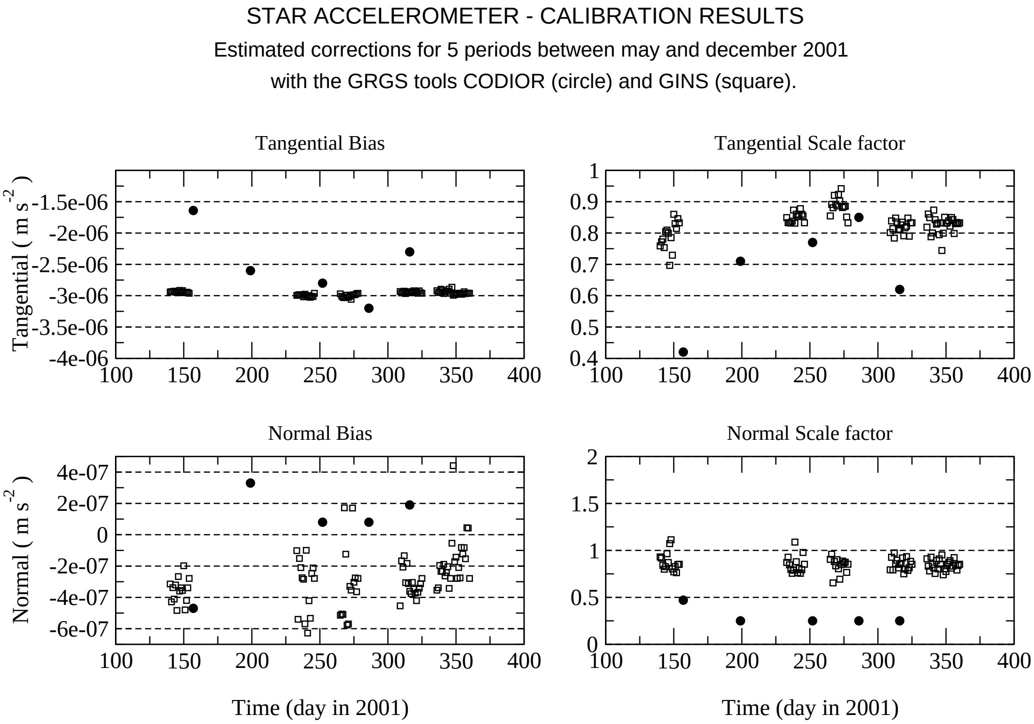

Fig. 2. Characteristics of the STAR accelerometer: comparison between GINS and CODIOR

a bias, before their use in the Gauss equations:

$$\left| \begin{array}{l} \boldsymbol{R}_{\text{real}} = C_R \boldsymbol{R}_{\text{STAR}} + B_R \\ \boldsymbol{T}_{\text{real}} = C_T \boldsymbol{T}_{\text{STAR}} + B_T \\ \boldsymbol{N}_{\text{real}} = C_N \boldsymbol{N}_{\text{STAR}} + B_N \end{array} \right. \tag{2}$$

Before the use of these measurements in the CODIOR orbit computation, the raw data (level 1 data from GFZ Postdam) have been filtered to reduce the noise, to eliminate outliers and to fill the data gaps. This numerical preprocessing is realized in three steps:

1. Filtering of the short periods smaller than the orbital period of about 5400 s in the raw data, thanks to a Vondrák filter (Vondrák, 1969; 1977),

2. Detection and removal of outliers by comparing raw and filtered data (3σ criterion),

3. Filling of the data gaps using spline interpolations. Only the gaps smaller than the orbital period can correctly be filled.

Due to problems on STAR, the data of the radial axe can not be used (Koenig et al., 2001). We suppose therefore $\boldsymbol{R} = 0$. Moreover, the biases and scale factors in the two other axes can be affected by very strong variations (in particular the biases), meaning the changes of the spatial environment of the satellite (magnetic storms for example). Nevertheless, they can be considered constant over the duration of a short arc (1 or 2 days).

What is at stake in a long arc computation is the definition of the mean characteristics of the accelerometer, i.e. only one value for each bias and each scale factor over a long period of time. A possible application of such an approach is the study of possible slow drift in these characteristics. With no uncontrolled jumps in the accelerometric data, this would be easy. But these jumps have a great influence on the mean motion. To manage them to a certain extent, we have empirically divided the considered period in five areas (see Table 2). For each area, we have adjusted the bias and the scale factor for the along track and accross track axes (i.e. 20 parameters over the whole period).

The results of these adjustments are shown in Table 2. They have been adjusted at the same time as the initial conditions of the arc. Figure 2 shows the comparison between the results derived by CODIOR and those derived by GINS, the GRGS classical software.

The results are quite similar, even if CODIOR adjusts so few empirical parameters. In particular, let note that the tangential bias has a better stability in GINS than in CODIOR. The difference of stability of the normal scale factor between the two softwares is still unexplained. A part of these differences could come from the difference of nature of the adjusted parameters in CODIOR and GINS. In this figure, for GINS, the plotted coefficients are C_T, C_N, $C_T\beta_T$, $C_N\beta_N$, since the parameters adjusted by GINS, C_T, C_N, β_T, β_N, verify:

$$\left| \begin{array}{l} \boldsymbol{R}_{\text{real}} = C_R(\boldsymbol{R}_{\text{STAR}} + \beta_R) \\ \boldsymbol{T}_{\text{real}} = C_T(\boldsymbol{T}_{\text{STAR}} + \beta_T) \\ \boldsymbol{N}_{\text{real}} = C_N(\boldsymbol{N}_{\text{STAR}} + \beta_N) \end{array} \right. \tag{3}$$

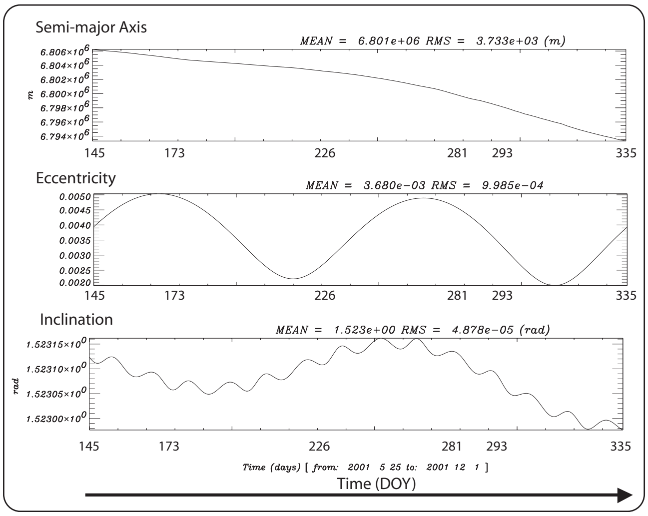

Fig. 3. Evolution of the metric elements of the long arc of CHAMP.

Nevertheless, we think that we can conclude that there is no slow drift in the accelerometer, drift which can not be seen with short arcs. A longer period of analysis would be useful to confirm that.

4 The Long Arc of CHAMP

On the basis of the available accelerometric data and of the "observed" mean elements, we have computed a mean arc of CHAMP over a period of 190 days in 2001, from May 25th to December 1st: since only the gaps in accelerometric data smaller than the orbital period can correctly be filled, the length of the mean arc computed with CODIOR is shorter than the period when "observed" mean elements are available, but reaches nevertheless more than the period of two revolutions of the argument of the perigee (about 92 days).

First, we have checked that CODIOR enables to compute a mean orbit as low as this of CHAMP: in Table 3, we show that the semi-analytical model built to compute mean orbits with an altitude of 800 km or more still leads to a great precision, and that the neglected coupling effects have not a strong influence at 400 kilometers.

Over that period of 190 days, we have computed a single long arc with the following standards:

– GRIM5-S1 for the gravity field model (Biancale et al., 2000),
– (Schwiderski, 1980) for the ocean tides,
– the Love numbers $k_2 = 0,299$ and $k_3 = 0,094$ to compute the terrestrial tides,
– the VSOP82 theory for the luni-solar effects and the planets (Bretagnon and Francou, 1988).

The shapes of the computed metric elements are shown Fig. 3, and the residuals in Fig. 4. In order to validate the averaging method of the accelerometric data, another arc

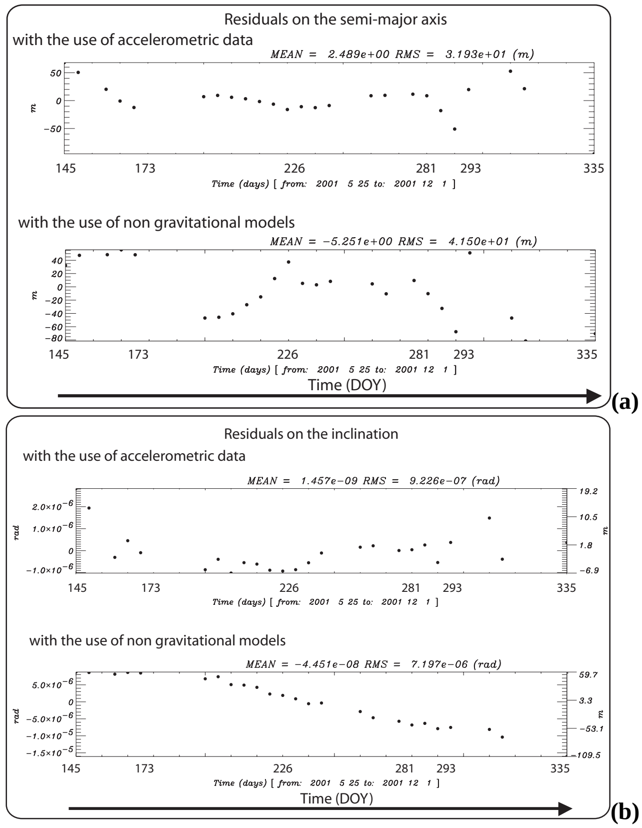

Fig. 4. Comparison of the results obtained with accelerometric data and with non gravitational models. The residuals we get on the semi-major axis are about 30-40 meters, and have to be compared to the total decrease due to the atmospheric drag which reaches 12 km. For the inclination, we get lower residuals when using accelerometric data, since the non-gravitational model we have used does not include the management of shadow effects. On the eccentricity, we get a level of residuals of $2,3\ 10^{-5}$ (which corresponds to 150 m, to be compared to a total variation of about 21 km). The residuals obtained on the angular elements are about $6,2\ 10^{-5}$ on the ascending node, $3,2\ 10^{-3}$ on the argument of the perigee, $1,2\ 10^{-2}$ on the rapid variable.

has been computed with a non-gravitational model including only the atmospheric drag (modelled in DTM-94; Berger et al., 1998). At the same time, the comparison with the first arc confirms that, at low altitudes, accelerometric data permit to reach a better accuracy of the orbit than non-gravitational models, even if the values of air drag scale factors are close to 1 (see Table 4).

5 Conclusion

Since few years, the concept of mean orbital motion is developed into the CANEL/CODIOR package, allowing to analyse the long periodic and secular variations affecting a long arc orbit (typically over several thousands of revolutions).

The mean gravitational model of the semi-analytical theory is actually suited for altitude greater than 800 km. Nev-

Table 3. Long period effects on the orbital parameters: Theoretical amplitudes due to gravitational effects, and corresponding residuals in the CODIOR software. Amplitudes of the long period effects depend on the altitude of the satellite. For each satellite, and for each mean variable a', i', Ω', C', S', λ', are shown these theoretical amplitudes due to gravitational effects (static part of the gravity field developed up to degree and order 40, terrestrial and oceanic tides, luni-solar effects) in the first column (not including the influence of J_2), and the level of residuals obtained in CODIOR in the second one. In each case, the residuals are not equal to zero because some negligible coupling effects are not included in the model [Deleflie,2002]. All quantities are expressed in meters (m), with arcs computed over a period of 200 days. It appears that the level of the residuals is still very satisfactory, even with a low altitude such as this of CHAMP.

	LAGEOS $\simeq 6000km$		STELLA $\simeq 800km$		CHAMP $\simeq 400km$	
	Ampl. of signal (m)	RMS (m)	Ampl. of signal (m)	RMS (m)	Ampl. of signal (m)	RMS (m)
a'	$4,7\ 10^{-3}$	$4,5\ 10^{-3}$	$9,6\ 10^{-3}$	$9,5\ 10^{-3}$	$5,3\ 10^{-3}$	$5,3\ 10^{-3}$
i'	$6,0\ 10^{3}$	$7,9\ 10^{-5}$	$3,8\ 10^{3}$	$6,6\ 10^{-1}$	$1,1\ 10^{3}$	$2,2\ 10^{-1}$
Ω'	$4,8\ 10^{3}$	$6,4\ 10^{-5}$	$4,5\ 10^{4}$	$2,0\ 10^{0}$	$2,0\ 10^{3}$	$2,0\ 10^{0}$
λ'	$1,3\ 10^{5}$	$2,5\ 10^{0}$	$1,9\ 10^{6}$	$7,6\ 10^{1}$	$3,8\ 10^{5}$	$4,4\ 10^{2}$
C'	$3,5\ 10^{3}$	$1,1\ 10^{-2}$	$1,4\ 10^{4}$	$4,3\ 10^{-2}$	$2,6\ 10^{3}$	$2,6\ 10^{0}$
S'	$1,2\ 10^{2}$	$3,6\ 10^{-9}$	$1,1\ 10^{3}$	$1,4\ 10^{-6}$	$1,1\ 10^{3}$	$5,7\ 10^{-4}$

Table 4. Values of air drag scale factors. Atmospheric drag model: DTM-94

	Limits of areas (DOY in 2001)	
Aera 1	from 143 to 173	1, 03
Area 2	from 173 to 226	1, 10
Area 3	from 226 to 281	0, 94
Area 4	from 281 to 293	1, 18
Area 5	from 293 to 335	0, 82

ertheless, an application to the CHAMP orbit has been realized here. Instead of mean non-gravitational models, we use accelerometric data provided by the STAR accelerometer. These are measured accelerations (of the total surface forces acting on the satellite), which are to be averaged during the long arc computation. Many difficulties have come from the use of these data, because of the data noise level, the numerous gaps in a long time-serie, and also because of the few parameters used in CODIOR to calibrate the measurements.

In this context, we have investigated the sensibility of the mean orbital motion of CHAMP to a long term variation (through the biases and scale factors) of the STAR accelerometer. The method is purely dynamical. We use the fact that analysing a single long arc of 200 days is much more efficient than using 1-day arcs, even if the later approach is much more precise, in terms of orbit determination, than the former.

Results of the long arc fitting thus allow to determine empirical coefficients with a level of relative precision of around 10^{-3}. This precision directly depends on the variations of the mean orbital elements. Obviously, in case of the CHAMP orbit, the semi-major axis variations, of about 12 km over a period of 200 days, are the greatest, allowing to compute along track coefficients with precision. In addition, the control of the decreasing of the satellite altitude is at the same relative level: the residual in the mean semi-major axis are around 30 m rms.

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